Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\therefore\ y=\frac{36\tan\delta}{\pi}\log\left[\frac{\sqrt{(1+p^2)}-\cos\delta}{\sqrt{(1+p^2)}+\cos\delta}\right]-\frac{36\tan\delta}{\pi}\log\tan^2\tfrac{1}{2}\delta.$$

From the two equations for x and y, we have

$$p^2+p\sqrt{(1+p^2)}=\sin\delta\cos\delta\tan\left[\frac{\pi x\cos\delta}{72}+\delta\right]-\sin^2\delta\ldots.(\mathrm{i}).$$

$$\sqrt{(1+p^2)}=\cos\delta\,\frac{\cot^2\frac{1}{2}\delta+e^{\pi y/36\tan\delta}}{\cot^2\frac{1}{2}\delta-e^{\pi y/36\tan\delta}}\ldots.(\mathrm{ii}).$$

Denote the second members of equations (i) and (ii) by X and Y. Then we have $p^2+p\sqrt{(1+p^2)}=X$, $\sqrt{(1+p^2)}=Y$. Whence, by eliminating p, we have

$$Y^2=\frac{(X+1)^2}{2X+1}\ldots.(\mathrm{iii}).$$

which is an equation expressing the general character of the curve, which is transcendental. Making $x=0$, we may find two values of y corresponding, which are evidently the starting and stopping points in the problem proposed.

When $x=0$, $X=0$. Hence from (iii) we find $Y=\pm1$, or

$$\cos\delta\,\frac{\cot^2\frac{1}{2}\delta+e^{\pi y/36\tan\delta}}{\cot^2\frac{1}{2}d-e^{\pi y/36\tan\delta}}=\pm1;$$

whence $e^{\pi y/36\tan\delta}=1$ or $\cot^4\frac{1}{2}\delta$.

$\therefore\ y=0$, or $\dfrac{144\tan\delta}{\pi}\log\cot\frac{1}{2}\delta$, and the distance from starting point to stopping point is the latter value of y.

For the latitude 36° 20′ we have for the distance from starting point to stopping point 37.56 miles.

MECHANICS.

144. Proposed by G. B. M. ZERR, A. M., Ph. D. Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Pressure is applied perpendicularly to the plane surface yz, bounding an otherwise infinite isotropic solid. Find the resultant displacements, if the pressure varies as $\sin\left(\dfrac{2\pi y}{a}\right)+\sinh\left(\dfrac{2\pi y}{a}\right)$.

Solution by the PROPOSER.

Measuring the axis of x perpendicularly to the plane face into the interior of the solid, the longitudinal stresses P, Q and the shearing stress T are functions of x and y only, and the general equations of internal equilibrium reduce to (Thompson and Tait, *Natural Philosophy*, Art. 697) the following:

$$\left.\begin{array}{l} dP/dx+dT/dy=0 \\ dT/dx+dQ/dy=0 \end{array}\right\}\ldots.(1)$$

Let α, β be the displacements of the point originally at (x, y), $k=$the elasticity of volume, and $n=$the elasticity of figure of the substance, and let $m=k+\frac{1}{3}n$, then

$$\left.\begin{array}{l} P=(m+n)d\alpha/dx+(m-n)d\beta/dy \\ Q=(m-n)d\alpha/dx+(m+n)d\beta/dy \\ T=n(d\beta/dx+d\alpha/dy) \end{array}\right\}\ldots.(2).$$

Let $\delta=d\alpha/dx+d\beta/dy=$the superficial dilation, and $\nabla^2=d^2/dx^2+d^2/dy^2$, then equations (1) become

$$\left.\begin{array}{l} md\delta/dx+n\nabla^2\alpha=0 \\ md\delta/dy+n\nabla^2\beta=0 \end{array}\right\}\ldots.(3).$$

Assume $P=\varphi(x)\left[\sin\left(\frac{2\pi y}{a}\right)+\sinh\left(\frac{2\pi y}{a}\right)\right]$

$$Q=\phi(x)\left[\sin\left(\frac{2\pi y}{a}\right)+\sinh\left(\frac{2\pi y}{a}\right)\right]$$

$$T=\theta(x)\left[\cos\left(\frac{2\pi y}{a}\right)+\cosh\left(\frac{2\pi y}{a}\right)\right].$$

From equations (1),

$$\varphi'(x)\left[\sin\left(\frac{2\pi y}{a}\right)+\sinh\left(\frac{2\pi y}{a}\right)\right]-\frac{2\pi}{a}\theta(x)\left[\sin\left(\frac{2\pi y}{a}\right)-\sinh\left(\frac{2\pi y}{a}\right)\right]=0.$$

$$\theta'(x)+\frac{2\pi}{a}\phi'(x)=0.$$

$$\therefore\ \theta(x)=\frac{a}{2\pi}\varphi'(x)\left[\frac{\sin\left(\frac{2\pi y}{a}\right)+\sinh\left(\frac{2\pi y}{a}\right)}{\sin\left(\frac{2\pi y}{a}\right)-\sinh\left(\frac{2\pi y}{a}\right)}\right]=\frac{a}{2\pi}B\varphi'(x),\text{ suppose.}$$

$$\therefore\ \theta'(x)=\frac{a}{2\pi}\varphi''(x)B.$$

$$\therefore\ \phi(x)[\sin(2\pi y/a)-\sinh(2\pi y/a)]+(a^2/4\pi^2)\varphi''(x)[\sin(2\pi y/a)$$

$$+\sinh(2\pi y/a)]=0,\text{ or }\phi(x)+B(a^2/4\pi^2)\varphi''(x)=0.$$

From (2), $d\alpha/dx=(P+Q)/4m+(P-Q)/4n,$
$d\beta/dy=(P+Q)/4m-(P-Q)/4n,$

$$\delta=\frac{1}{2m}(P+Q)=\frac{1}{2m}[\varphi(x)+\phi(x)][\sin(2\pi y/a)+\sinh(2\pi y/a)].$$

If $P=pF(y)$ represent the distribution of traction and pressure on the plane face, then $\delta=P/m=(p/m)F(y)$, when $x=0$, and from equations (3). $\nabla^2\delta=0$; δ must also vanish when $x=\infty$.

$$\therefore\ \nabla^2\delta=(1/2m)\{[\varphi''(x)+\psi''(x)][\sin(2\pi y/a)+\sinh(2\pi y/a)]$$
$$-(4\pi^2/a^2)[\varphi(x)+\phi(x)][\sin(2\pi y/a)-\sinh(2\pi y/a)]\}=0.$$

$$\therefore\ B[\varphi''(x)+\phi''(x)]-(4\pi^2/a^2)[\varphi(x)+\phi(x)]=0.$$

$$\therefore B[\varphi''(x)-B(a^2/4\pi^2)\varphi''''(x)]-(4\pi^2/a^2)[\varphi(x)-B(a^2/4\pi^2)\varphi''(x)]=0.$$

$$\therefore\ 2B\varphi''(x)-(4\pi^2/a^2)\varphi(x)-B^2(a^2/4\pi^2)\varphi''''(x)=0,$$

Let $\varphi(x)+\phi(x)=\varphi(x)-B(a^2/4\pi^2)\varphi''(x)=f(x).$

$\therefore\ Bf''(x)-(4\pi^2/a^2)f(x)=0.\quad \therefore f(x)=Ce^{-(2\pi x/a\sqrt{B})}.$

Let $2\pi/a\sqrt{B}=b.\quad \therefore\ \varphi''(x)-b^2\varphi(x)+b^2Ce^{-bx}=0.$

The solution of this equation is $\varphi(x)=De^{-bx}-\frac{1}{2}bCxe^{-bx}.$

$$\therefore\ \theta(x)=-\frac{abB}{2\pi}De^{-bx}-\frac{abB}{4\pi}Ce^{-bx}+\frac{ab^2B}{4\pi}Cxe^{-bx}.$$

$$\psi(x)=-De^{-bx}-Ce^{-bx}+\tfrac{1}{2}bCxe^{-bx}.$$

Now when $x=0$, $P=Q=p[\sin(2\pi y/a)+\sinh(2\pi y/a)]$. $T=0$; therefore $\varphi(0)=\phi(0)=p$, $\theta(0)=0$; $\therefore$ $D=p$ and $C+2D=0$ or $C=-2p$.

$$\therefore\ P=pe^{-bx}(1+bx)[\sin(2\pi y/a)+\sinh(2\pi y/a)].$$
$$Q=pe^{-bx}(1-bx)[\sin(2\pi y/a)+\sinh(2\pi y/a)].$$
$$T=-\frac{2\pi dx}{a}e^{-bx}[\cos(2\pi y/a)+\cosh(2\pi y/a)].$$
$$\delta=(p/m)e^{-bx}[\sin(2\pi y/a)+\sinh(2\pi y/a)].$$

$$d\alpha/dx=\tfrac{1}{2}pe^{-bx}(1/m+bx/n)[\sin(2\pi y/a)+\sinh(2\pi y/a)].$$
$$d\beta/dy=\tfrac{1}{2}pe^{-bx}(1/m-bx/n)[\sin(2\pi y/a)+\sinh(2\pi y/a)].$$
$$\alpha=-(p/2b)e^{-bx}[1/m+1/n(1+bx)][\sin(2\pi y/a)+\sinh(2\pi y/a)].$$
$$\beta=\tfrac{1}{2}p\int e^{-bx}(1/m-bx/n)[\sin(2\pi y/a)+\sinh(2\pi y/a)].$$

In this last, $b=\frac{2\pi}{a}\sqrt{\frac{\sin(2\pi y/a)-\sinh(2\pi y/a)}{\sin(2\pi y/a)+\sinh(2\pi y/a)}}.$